DECISION MAKING APPROACH ON NORMAL SPASMODIC BIPOLAR FUZZY DELTA SOFT SUBGROUPS VIA LEVEL CUTSETS

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Abstract: In this article, the notion of spasmodic fuzzy soft sets, the idea of a spasmodic fuzzy soft groupoid in a given set δ and related properties are investigated. Spasmodic bipolar fuzzy soft groupoid is also constructed by using spasmodic bipolar fuzzy soft groupoid. Conversely spasmodic fuzzy soft groupoid is established by the way of spasmodic bipolar fuzzy soft groupoid. The characterisations of spasmodic bipolar fuzzy soft groupoid are provided and normal spasmodic bipolar fuzzy soft groupoid are discussed. Finally the decision making approach for spasmodic bipolar fuzzy soft set is to be analysed with a suitable example.

Keywords: Fuzzy soft set, bipolar fuzzy, spasmodic fuzzy set, normal, associative, Demorgan's Law, decision making, algorithms, rank.

1. Introduction: In 1999, soft set theory was introduced by Molodtsov [12] as an alternative approach to fuzzy soft set theory, defined by Zadeh [17] . A new structure called the soft inter- group was defined by Cagman et.al [3] and some properties of this new structure was obtained. A ring structure on soft sets was constructed by Acar et.al [2]. The soft inter-group was constructed by Kaygisiz [5] and the normal soft inter-group was defined and some properties were investigated. A traditional fuzzy set is characterized by the membership function, whose range is in the unit interval of [0, 1]. These are the several kinds of fuzzy extensions in the fuzzy set theory, for example Intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets etc. Bipolar fuzzy set was introduced by Zhang [[15],[16]] as a generalization of the fuzzy set. Bipolar fuzzy set is an extension of the fuzzy set whose membership degree interval is [-1,1].Abdullah et.al[1] introduced the notion of bipolar fuzzy soft sets, combining soft sets and bipolar fuzzy soft sets and has also defined the operations of bipolar fuzzy soft sets. Kim et.al [9] has studied, the ideal theory of semi groups based on the bipolar valued fuzzy set. Fuzzy sub semi groups and fuzzy ideals, with operators in semi groups are defined by Hur et.al [6].Therefore the notions of bipolar soft sets and their operations were defined by Koraaslam [[4],[7]] and the concept of bipolar fuzzy soft -semi group and bipolar fuzzy soft-ideals in semi groups.

2. Preliminaries and Basic Laws

In this section, we have discussed the basic idea and the elementary properties are explained.

2.1 Definition: Let U be an non empty finite set of objects called Universe and let E be a non empty parameters. An ordered pair (F, E)is said to be a soft set over U, if F is a mapping from E into the set of all subsets of U. That is $F : E \to P(U)$

It has been interpreted that a soft set indeed is a parameterized family of subset of U.

2.2 Example: Let $U = \{x_1, x_2, x_3\}$be the set of three phones and E= {size (y_1), colour (y_2), rate (y_3)} be the set of parameters where $A = \{y_1, y_2\} \subset E$.

Then $(F, A) = \{F(y_1) = \{x_1, x_2, x_3\}, F(y_2) = \{x_1, x_3\}$is the crisp soft set over U which describes the "attractiveness of the phones" which Mr. X (say) is going to buy.

2.3 Definition: Let G be the Universe of discourse, A bipolar fuzzy set A in G is an object having the form $A = \{(x, A_P(x), A_N(x)) / x \in G\}$ where $A_P : G \to [0,1]$ and $A_N : G \to [-1,0]$ are mappings. The positive membership degree $A_P(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set A and the negative membership degree $A_N(x)$ denotes the satisfaction degree of x to some implicit counter property of A. It is possible for an element 'x'to be $A_P(x) \neq 0$ and $A_N(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of the domain. By simplification, use the symbol $A = (G; A_P, A_N)$where as the bipolar fuzzy set $A = \{(x, A_P(x), A_N(x)) / x \in G\}$by a groupoid of a group G, we mean the non empty subset B of G, such that $B^2 \subset B$.

2.4 Definition: A fuzzy set in G is called a fuzzy groupoid of G, if it satisfies $\mu(xy) \geq \min\{\mu(x), \mu(y)\} \ \forall \ x, y \in G\}$. For any family $\{a_i / i \in \Omega\}$ of real numbers, we define

$$\max\{a_i / i \in \Omega\} = \begin{cases} \max\{a_i / i \in \Omega\} & \text{if } \Omega \text{ is finite} \\ \sup\{a_i / i \in \Omega\} & \text{otherwise} \end{cases}$$

$$\min\{a_i / i \in \Omega\} = \begin{cases} \min\{a_i / i \in \Omega\} & \text{if } \Omega \text{ is finite} \\ \inf\{a_i / i \in \Omega\} & \text{otherwise} \end{cases}$$

2.5 Definition: A spasmodic bipolar fuzzy set $A_N^{\delta}(xy, \alpha) \leq \max\{A_N^{\delta}(x, \alpha), A_N^{\delta}(y, \alpha)\}$ in G is called a spasmodic fuzzy soft groupoid Gif it satisfies the following conditions $= 0.4$ and $A_N(xy) \leq \max\{A_N(x), A_N(y)\} \ \forall \ x, y \in G$.

In what follows, Let G and δ denote a group and a non empty set respectively, unless otherwise specified.

2.6 Definition: A spasmodic bipolarfuzzy soft set (SBFSS) $A_{\delta} = \langle G \times \delta; A_P^{\delta}, A_N^{\delta} \rangle$ in G is called a spasmodic bipolarfuzzy soft groupoids of G if it satisfies $A_P^{\delta}(xy, \alpha) \geq \min\{A_P^{\delta}(x, \alpha), A_P^{\delta}(y, \alpha)\}$

and $A_N^{\delta}(xy, \alpha) \leq \max\{A_N^{\delta}(x, \alpha), A_N^{\delta}(y, \alpha)\} \ \forall \ x, y \in G$ *and* $\alpha \in \delta$.

2.7 Example: Consider a group $G = \{l, m\}$ with the following Cayley table

.	l	m
l	l	m
m	m	l

Let $\delta = \{1,2\}$ and let $A_\delta = \{G \cdot \delta; A_P^\delta, A_N^\delta\}$ be a spasmodic bipolar fuzzy soft set in G defined by,

$A_\delta = \{< (l,1);-0.4, 1 >, < (l,2);-0.9, 1 >, < (m,1); -0.7, 0.4 >, < (m,2); -0.7, 0.8 >\}$.It is easy to verify that $A_\delta = \{G \cdot \delta; A_P^\delta, A_N^\delta\}$ is a spasmodicbipolar fuzzy soft groupoids of G.

3. Basic Laws of spasmodic bipolar fuzzy soft sets

3.1 Proposition (Demorgan's law): Let $A_\delta, B_\delta \in BP\delta\, FSS$ then prove that $(A_\delta \cap B_\delta)^C = A_\delta^C \cup B_\delta^C$

Proof: Let $x \in G$ and $\alpha \in \delta$

LHS:

$$(A_\delta \cap B_\delta)^C (x,\alpha) = 1-(A_\delta \cap B_\delta)(x,\alpha)$$
$$= 1-\max\{A_\delta(x,\alpha), B_\delta(x,\alpha)\}$$
$$= \min\{1-A_\delta(x,\alpha), 1-B_\delta(x,\alpha)\}$$
$$= \min\{A_\delta^C(x,\alpha), B_\delta^C(x,\alpha)\}$$

RHS:

$$(A_\delta^C \cup B_\delta^C)(x,\alpha) = ((1-A_\delta) \cup (1-B_\delta))(x,\alpha)$$
$$= (1-A_\delta(x,\alpha)) \cup (1-B_\delta(x,\alpha))$$
$$= \min\{1-A_\delta(x,\alpha), 1-B_\delta(x,\alpha)\}$$
$$= \min\{A_\delta^C(x,\alpha), 1-B_\delta^C(x,\alpha)\}$$

$\therefore$ LHS = RHS

3.2 Proposition (Distributive law): Let $A_\delta, B_\delta, C_\delta \in BP\delta\, FSS$. Then the following conditions on hold,

(i) $A_\delta \cup (B_\delta \cap C_\delta) = (A_\delta \cup B_\delta) \cap (A_\delta \cup C_\delta)$

(ii) $A_\delta \cap (B_\delta \cup C_\delta) = (A_\delta \cap B_\delta) \cup (A_\delta \cap C_\delta)$

Proof: Let $x, y \in G$ and $\alpha \in \delta$

LHS:

$$A_\delta \cup (B_\delta \cap C_\delta)(x,\alpha) = \min\{A_\delta(x,\alpha), (B_\delta \cap C_\delta)(x,\alpha)\}$$
$$= \max\{\min\{A_\delta(x,\alpha), B_\delta(x,\alpha)\}, \min\{A_\delta(x,\alpha), C_\delta(x,\alpha)\}$$
$$= \max\{(A_\delta \cup B_\delta)(x,\alpha), (A_\delta \cup C_\delta)(x,\alpha)\}$$
$$= (A_\delta \cup B_\delta) \cap (A_\delta \cup C_\delta)(x,\alpha)$$

RHS:

$$(A_\delta \cup B_\delta) \cap (A_\delta \cup C_\delta)(x,\alpha) = \max\{(A_\delta \cup B_\delta)(x,\alpha), (A_\delta \cup C_\delta)(x,\alpha)\}$$
$$= \max\{\min\{A_\delta(x,\alpha), B_\delta(x,\alpha)\}, \min\{A_\delta(x,\alpha), C_\delta(x,\alpha)\}$$
$$= \min\{\max\{A_\delta(x,\alpha), B_\delta(x,\alpha)\}, \max\{A_\delta(x,\alpha), C_\delta(x,\alpha)\}$$
$$= \min\{\max\{A_\delta(x,\alpha)\}, \{B_\delta(x,\alpha), C_\delta(x,\alpha)\}$$
$$= \min\{A_\delta(x,\alpha), \max\{B_\delta(x,\alpha), C_\delta(x,\alpha)\}$$
$$= A_\delta \cup (B_\delta \cap C_\delta)(x,\alpha)$$

$\therefore$ LHS = RHS

Similarly we can show (ii).

3.3 Proposition (Complementary Law): Let $A_\delta, B_\delta \in BP\delta\ FSS$. Then the following conditions on hold

(i) $(A_\delta \cup B_\delta)^C = A_\delta^{\ C} \cap B_\delta^{\ C}$

(ii) $(A_\delta \cap B_\delta)^C = A_\delta^{\ C} \cup B_\delta^{\ C}$

Proof: Let $x \in G$ and $\alpha \in \delta$

$$(A_\delta \cup B_\delta)^C(x,\alpha) = 1 - (A_\delta \cup B_\delta)(x,\alpha)$$
$$= 1 - \min\{A_\delta(x,\alpha), B_\delta(x,\alpha)\}$$
$$= \max\{1 - A_\delta(x,\alpha), 1 - B_\delta(x,\alpha)\}$$
$$= \max\{A_\delta^{\ C}(x,\alpha), B_\delta^{\ C}(x,\alpha)\}$$

$$(A_\delta^{\ C} \cap B_\delta^{\ C})(x,\alpha) = ((1 - A_\delta) \cap (1 - B_\delta))(x,\alpha)$$
$$= \max\{1 - A_\delta(x,\alpha), 1 - B_\delta(x,\alpha)\}$$
$$= \max\{A_\delta^{\ C}(x,\alpha), B_\delta^{\ C}(x,\alpha)\}$$

$\therefore$ LHS = RHS. Similarly we can show (ii).

3.4 Proposition : (Associative Law): Let $A_\delta, B_\delta \in BP\delta\ FSS$. Then prove

(i) $A_\delta \cup (B_\delta \cup C_\delta) = (A_\delta \cup B_\delta) \cup C_\delta$

(ii) $A_\delta \cap (B_\delta \cap C_\delta) = (A_\delta \cap B_\delta) \cap C_\delta$

Proof: $(A_\delta \cup (B_\delta \cup C_\delta))(x,\alpha) = \min\{A_\delta(x,\alpha), (B_\delta \cup C_\delta)(x,\alpha)\}$

$$= \min\{A_\delta(x,\alpha), \min\{B_\delta(x,\alpha), C_\delta(x,\alpha)\}$$

$$= \min\{\min\{A_\delta(x,\alpha), B_\delta(x,\alpha)\}, C_\delta(x,\alpha)\}$$

$$= \min\{(A_\delta \cup B_\delta)(x,\alpha), C_\delta(x,\alpha)\}$$

$$= (A_\delta \cup B_\delta)(x,\alpha) \cup C_\delta(x,\alpha)$$

$$= ((A_\delta \cup B_\delta) \cup C_\delta)(x,\alpha)$$

$$= (A_\delta \cup B_\delta) \cup C_\delta$$

Hence the proof. Similarly we can show (ii).

3.5 Theorem : Let δ be the set of all spasmodic bipolar fuzzy soft groupoid of G and let $A_\delta = \{G \cdot \delta; A_P^\delta, A_N^\delta\}$ be a spasmodic fuzzy soft sets in G when $A_N^\delta(x, A) = A_N(x)$ and $A_P^\delta(x, A) = A_P(x)$ for $x \in G$ and $A = (G; A_P, A_N)$. Then $A_\delta = \{G \cdot \delta; A_P^\delta, A_N^\delta\}$ is a spasmodic bipolar fuzzy soft groupoid of G.

Proof: Let $x, y \in G$ and $A = (G; A_P, A_N)$. Then $A_P^\delta(xy, A) = A_P(xy) \cdot \min\{A_P(x), A_P(y)\} = \min\{A_P^\delta(x), A_P^\delta(y)\}$ and $A_N^\delta(xy, A) = A_N(xy) \cdot \max\{A_N(x), A_N(y)\} = \max\{A_N^\delta(x), A_N^\delta(y)\}$.

Hence $A_\delta = \{G \cdot \delta; A_P^\delta, A_N^\delta\}$ is a spasmodic bipolar fuzzy soft groupoid of G.

3.6 Theorem : If $A_\delta = \{G \cdot \delta; A_P^\delta, A_N^\delta\}$ is a spasmodic bipolar fuzzy soft groupoid of G and $\alpha \in \delta$, then a spasmodic soft set $A = \{G; A_P^\alpha, A_N^\alpha\}$ where $A_P^\delta : G \to [0,1]$, $x \to A_P^\delta(x,\alpha)$ and $A_N^\delta : G \to [-1,0]$, $x \to A_N^\delta(x,\alpha)$ is a spasmodic bipolar fuzzy soft groupoid of G.

Proof: Let $x, y \in G$. Then $A_P^\delta(xy,\alpha) = A_P^\delta(xy,\alpha) \cdot \min\{A_P^\delta(x,\alpha), A_P^\delta(y,\alpha)\} = \min\{A_P^\alpha(x), A_P^\alpha(y)\}$ and $A_N^\delta(xy,\alpha) = A_N^\delta(xy,\alpha) \cdot \max\{A_N^\delta(x,\alpha), A_N^\delta(y,\alpha)\} = \max\{A_N^\alpha(x), A_N^\alpha(y)\}$. This completes the proof.

3.7 Theorem : If $A = \{G; A_P^\alpha, A_N^\alpha\}$, $\alpha \in \delta$ is a spasmodic bipolar fuzzy soft groupoid of G, then a spasmodic fuzzy soft set $A_\delta = \{G \cdot \delta; A_P^\delta, A_N^\delta\}$ where $A_P^\delta : G \cdot \delta \to [0,1]$, $(x,\alpha) \to A_P^\alpha(x)$ and $A_N^\delta : G \cdot \delta \to [-1,0]$, $(x,\alpha) \to A_N^\alpha(x)$ is a spasmodic bipolar fuzzy soft groupoid of G.

Proof: For any $x, y \in G$, we have $A_P^\delta(xy,\alpha) = A_P^\alpha(xy) \cdot \min\{A_P^\alpha(x), A_P^\alpha(y)\} = \min\{A_P^\delta(x,\alpha), A_P^\delta(y,\alpha)\}$ and $A_N^\delta(xy,\alpha) = A_N^\alpha(xy) \cdot \max\{A_N^\alpha(x), A_N^\alpha(y)\} = \min\{A_N^\delta(x,\alpha), A_N^\delta(y,\alpha)\}$.

Hence A_δ is a spasmodic bipolar fuzzy soft groupoid.

3.8 Theorem: Let $f_\delta = (G^\delta : f_P{}^\delta, f_N{}^\delta)$ be a spasmodic fuzzy bipolar soft groupoid of G^δ and let $A_\delta = \left\{G \cdot \delta; A_P^\delta, A_N^\delta\right\}$ be a spasmodic bipolar fuzzy soft set in G define by $A_P{}^\delta(x,\alpha) = \max\left\{f_P^\delta(u)/u \in G^\delta, u(\alpha) = x\right\}$, $A_N^\delta(x,\alpha) = \min\left\{f_N^\delta(u)/u \in G^\delta, u(\alpha) = x\right\}$for all $x \in G$ and $\alpha \in \delta$. Then $A_\delta = \left\{G \cdot \delta; A_P{}^\delta, A_N{}^\delta\right\}$is a spasmodic bipolar fuzzy soft groupoid in G.

Proof: Let $x, y \in G$ and $\alpha \in \delta$. Then

$$\begin{aligned} A_P{}^\delta(xy,\alpha) &= \max\left\{f_P{}^\delta(u)/u \in G^\delta, u(\alpha) = xy\right\} \\ &\cdot \max\left\{f_P{}^\delta(uv)/u, v \in G^\delta, u(\alpha) = x, v(\alpha) = y\right\} \\ &\cdot \max\left\{\min\left\{f_P{}^\delta(u), f_P{}^\delta(v)\right\}/u, v \in G^\delta, u(\alpha) = x, v(\alpha) = y\right\} \\ &= \min\left\{\max\left\{\{f_P{}^\delta(u)/u \in G^\delta, u(\alpha) = x\}, \{f_P{}^\delta(v)/v \in G^\delta, v(\alpha) = y\}\right\}\right\} \\ &= \min\left\{f_P{}^\delta(x,\alpha), f_P{}^\delta(y,\alpha)\right\} \end{aligned}$$

and

$$\begin{aligned} A_N{}^\delta(xy,\alpha) &= \min\left\{f_N{}^\delta(u)/u \in G^\delta, u(\alpha) = xy\right\} \\ &\cdot \min\left\{f_N^\delta(uv)/u, v \in G^\delta, u(\alpha) = x, v(\alpha) = y\right\} \\ &\cdot \min\left\{\max\left\{f_N{}^\delta(u), f_N{}^\delta(v)/u, v \in G^\delta, u(\alpha) = x, v(\alpha) = y\right\}\right\} \\ &= \max\left\{\min\left\{\left\{f_N{}^\delta(u)/u \in G^\delta, u(\alpha) = x\right\}, \left\{f_N{}^\delta(v)/v \in G^\delta, v(\alpha) = y\right\}\right\}\right\} \\ &= \max\left\{f_N{}^\delta(x,\alpha), f_N{}^\delta(y,\alpha)\right\} \end{aligned}$$

Hence A_δ is a spasmodic bipolar fuzzy soft groupoid of G.

3.9 Example: Let $G = \{l, m\}$be a group in example 4.2.7 and let$\delta = \{1,2\}$. Then$G^\delta = \{e, a, b, c\}$ when $e(1) = e(2) = b(1) = b(2) = c(2) = l$ and $a(1) = a(2) = b(2) = c(2) = m$ is a group (a commutative group) under the following Cayley table

*	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Let $f_\delta = (G^\delta; f_P{}^\delta, f_N{}^\delta)$ be a spasmodic bipolar fuzzy soft set in G^δ defined by $f_\delta = \{(e; -0.7, 0.6), (a; -0.4, 0.2), (b; -0.9, 0.5), (c; -0.9, 0.5)\}$. Then f_δ is a spasmodic bipolar fuzzy soft groupoid of G^δ, Thus we can obtain a spasmodic bipolarfuzzy soft groupoid $A_\delta = \left\{G \cdot \delta; A_P^\delta, A_N^\delta\right\}$of Gas follows

$$A_P{}^\delta(l,1) = \max\left\{f_P{}^\delta(u)/u \in G^\delta, u(1) = l\right\} = \max\left\{f_P{}^\delta(e), f_P{}^\delta(b)\right\} = 0.6$$

$$A_P{}^\delta(l,2) = \max\left\{f_P{}^\delta(u)/u \in G^\delta, u(2) = l\right\} = \max\left\{f_P{}^\delta(e), f_P{}^\delta(c)\right\} = 0.6$$

$$A_P{}^{\delta}(m,1) = \max\left\{f_P{}^{\delta}(u)/u \in G^{\delta}, u(1) = m\right\} = \max\left\{f_P{}^{\delta}(a), f_P{}^{\delta}(c)\right\} = 0.5$$

$$A_P{}^{\delta}(m,2) = \max\left\{f_P{}^{\delta}(u)/u \in G^{\delta}, u(2) = m\right\} = \max\left\{f_P{}^{\delta}(a), f_P{}^{\delta}(b)\right\} = 0.5$$

$$A_N{}^{\delta}(l,1) = \min\left\{f_N{}^{\delta}(v)/v \in G^{\delta}, v(1) = l\right\} = \min\left\{f_N{}^{\delta}(e), f_N{}^{\delta}(b)\right\} = -0.9$$

$$A_N{}^{\delta}(l,2) = \min\left\{f_N{}^{\delta}(v)/v \in G^{\delta}, v(2) = l\right\} = \min\left\{f_N{}^{\delta}(e), f_N{}^{\delta}(c)\right\} = -0.9$$

$$A_N{}^{\delta}(m,1) = \min\left\{f_N{}^{\delta}(v)/v \in G^{\delta}, v(1) = m\right\} = \min\left\{f_N{}^{\delta}(a), f_N{}^{\delta}(c)\right\} = -0.9$$

$$A_N{}^{\delta}(m,2) = \min\left\{f_N{}^{\delta}(v)/v \in G^{\delta}, v(2) = m\right\} = \min\left\{f_N{}^{\delta}(a), f_N{}^{\delta}(b)\right\} = -0.9$$

3.10 Theorem : Let A_{δ} be a spasmodic bipolar fuzzy soft groupoid of G and let $f_{\delta} = (G^{\delta}; f_P{}^{\delta}, f_N{}^{\delta})$ be a spasmodic bipolar fuzzy soft set in G^{δ} define by $f_P{}^{\delta}(u) = \min\left\{A_P{}^{\delta}(u(\alpha),\alpha)/\alpha \in \delta\right\}$ and $f_N{}^{\delta}(u) = \max\left\{A_N{}^{\delta}(u(\alpha),\alpha)/\alpha \in \delta\right\}$for all $u \in G^{\delta}$. Then f_{δ} is a spasmodic bipolar fuzzy soft groupoid of G^{δ}.

Proof: For any $u, v \in G^{\delta}$, we have

$$\begin{aligned} f_P{}^{\delta}(uv) &= \min\left\{A_P{}^{\delta}((uv)(\alpha),\alpha)/\alpha \in \delta\right\} \\ &= \min\left\{A_P{}^{\delta}\left(u(\alpha)v(\alpha),\alpha\right)/\alpha \in \delta\right\} \\ &\cdot \min\left\{\min\left\{A_P{}^{\delta}\left(u(\alpha),\alpha\right)/\alpha \in \delta\right\}, \min\left\{A_P{}^{\delta}\left(v(\alpha),\alpha\right)/\alpha \in \delta\right\}\right\} \\ &= \min\left\{f_P{}^{\delta}(u), f_P{}^{\delta}(v)\right\} \text{and} \end{aligned}$$

$$\begin{aligned} f_N{}^{\delta}(uv) &= \max\left\{A_N{}^{\delta}((uv)(\alpha),\alpha)/\alpha \in \delta\right\} \\ &= \max\left\{A_N{}^{\delta}\left(u(\alpha)v(\alpha),\alpha\right)/\alpha \in \delta\right\} \\ &\cdot \max\left\{\max\left\{A_N{}^{\delta}\left(u(\alpha),\alpha\right)/\alpha \in \delta\right\}, \max\left\{A_N{}^{\delta}\left(v(\alpha),\alpha\right)/\alpha \in \delta\right\}\right\} \\ &= \max\left\{f_N{}^{\delta}(u), f_N{}^{\delta}(v)\right\} \end{aligned}$$

Thus $f_{\delta} = (G^{\delta}; f_P{}^{\delta}, f_N{}^{\delta})$ be a spasmodic bipolar fuzzy soft groupoid of G^{δ}.

3.11 Example: $A_{\delta} = \left\{G \cdot \delta; A_P^{\delta}, A_N^{\delta}\right\}$ be a spasmodic bipolar fuzzy soft groupoid. We assume G^{δ} is a commutative group in example 2.7 compare with example 3.9.Then we can induce a spasmodic bipolar fuzzy soft groupoid for $(G^{\delta}; f_P{}^{\delta}, f_N{}^{\delta})$ of G^{δ} as follows

Positive	Negative
$f_P{}^{\delta}(e) = \min\left\{A_P{}^{\delta}(e(\alpha),\alpha)/\alpha \in \delta\right\}$ $= \min\left\{A_P{}^{\delta}(e(1),1), A_P{}^{\delta}(e(2),2)\right\}$ $= \min\left\{A_P{}^{\delta}(l,1), A_P{}^{\delta}(l,2)\right\} = 0.6$ $f_P{}^{\delta}(a) = \min\left\{A_P{}^{\delta}(a(\alpha),\alpha)/\alpha \in \delta\right\}$ $= \min\left\{A_P{}^{\delta}(a(1),1), A_P{}^{\delta}(a(2),2)\right\}$ $= \min\left\{A_P{}^{\delta}(m,1), A_P{}^{\delta}(m,2)\right\} = 0.3$	$f_N{}^{\delta}(e) = \max\left\{A_N{}^{\delta}(e(\alpha),\alpha)/\alpha \in \delta\right\}$ $= \max\left\{A_N{}^{\delta}(e(1),1), A_N{}^{\delta}(e(2),2)\right\}$ $= \max\left\{A_N{}^{\delta}(l,1), A_N{}^{\delta}(l,2)\right\} = -0.9$ $f_N{}^{\delta}(a) = \max\left\{A_N{}^{\delta}(a(\alpha),\alpha)/\alpha \in \delta\right\}$ $= \max\left\{A_N{}^{\delta}(a(1),1), A_N{}^{\delta}(a(2),2)\right\}$

$f_P^{\delta}(b) = \min\left\{A_P^{\delta}(b(\alpha),\alpha)/\alpha\in\delta\right\}$	$= \max\left\{A_N^{\delta}(m,1), A_N^{\delta}(m,2)\right\} = -0.9$
$= \min\left\{A_P^{\delta}(b(1),1), A_P^{\delta}(b(2),2)\right\}$	$f_N^{\delta}(b) = \max\left\{A_N^{\delta}(b(\alpha),\alpha)/\alpha\in\delta\right\}$
$= \min\left\{A_P^{\delta}(l,1), A_P^{\delta}(m,2)\right\} = 0.5$	$= \max\left\{A_N^{\delta}(b(1),1), A_N^{\delta}(b(2),1)\right\}$
$f_P^{\delta}(c) = \min\left\{A_P^{\delta}(c(\alpha),\alpha)/\alpha\in\delta\right\}$	$= \max\left\{A_N^{\delta}(l,1), A_N^{\delta}(m,1)\right\} = -0.9$
$= \min\left\{A_P^{\delta}(c(1),1), A_P^{\delta}(c(2),2)\right\}$	$f_N^{\delta}(c) = \max\left\{A_N^{\delta}(c(\alpha),\alpha)/\alpha\in\delta\right\}$
$= \min\left\{A_P^{\delta}(l,2), A_P^{\delta}(m,1)\right\} = 0.3$	$= \max\left\{A_N^{\delta}(c(1),1), A_N^{\delta}(c(2),2)\right\}$
	$= \max\left\{A_N^{\delta}(l,2), A_N^{\delta}(m,2)\right\} = -0.9$

For a spasmodic bipolar fuzzy soft set A_δ in G and $(s,t)\in[-1,0]\times[0,1]$, we define $P(A_\delta;t) = \left\{x\in G / A_P^{\delta}(x,\alpha) \geq t \;\; \forall\, \alpha\in\delta\right\}$, $N(A_\delta;s) = \left\{x\in G / A_N^{\delta}(x,\alpha) \leq s \;\; \forall\, \alpha\in\delta\right\}$

Which are called the positive t-cut of A_δ and negative s-cut of A_δ respectively. The set $D(A_\delta;(s,t)) = P(A_\delta;t)\cap N(A_\delta;s)$ is called (s,t)-cut of A_δ. For every $k\in[0,1]$, if $(s,t) = (-k,k)$, then the set $D(A_\delta;k) = P(A_\delta;k)\cap N(A_\delta;-k)$ is called the k-cut of A_δ.

3.12 Theorem : Let a spasmodic bipolar fuzzy soft set A_δ in G is a spasmodic bipolar fuzzy soft groupoid of G.Then the following assumptions are valid:

(i) $P(A_\delta;t)\neq\phi \Rightarrow P(A_\delta;t)$ is a soft groupoid of G $\forall\, x\in[0,1]$

(ii) $N(A_\delta;s)\neq\phi \Rightarrow N(A_\delta;s)$ is a soft groupoid of G $\forall\, x\in[-1,0]$

Proof:

(i) Now, let $t\in[0,1]$ be such that $P(A_\delta;t)\neq\phi$ if $x,y\in P(A_\delta;t)$, then $A_P^{\delta}(x,\alpha)\geq t$ and $A_P^{\delta}(y,\alpha)\geq t$ for $\alpha\in\delta$ and so $A_P^{\delta}(xy,\alpha)\geq\min\left\{A_P^{\delta}(x,\alpha), A_P^{\delta}(y,\alpha)\right\}\geq t$.

Hence $P(A_\delta;t)$ is a soft groupoid of G.

(ii) Let $s\in[-1,0]$ be such that $N(A_\delta;s)\neq\phi$ if $x,y\in N(A_\delta;s)$, then $A_N^{\delta}(x,\alpha)\leq s$ and $A_N^{\delta}(y,\alpha)\leq s$ for $\alpha\in\delta$. It follows that $A_N^{\delta}(xy,\alpha)\leq\max\left\{A_N^{\delta}(x,\alpha), A_N^{\delta}(y,\alpha)\right\}\leq s$.

Hence $N(A_\delta;s)$ is a soft groupoid of G.

3.13 Theorem : Let A_δ be a bipolar fuzzy soft set in G satisfying two conditions (i) and (ii) in theorem 2.6.Then A_δ is a spasmodic bipolar fuzzy soft groupoid of G.

Proof: Assume that A_δ is not a spasmodic bipolar fuzzy soft groupoid of G. Then the condition is false. (ie) there exist $l,m\in G$ and $\alpha\in\delta$ such that

$A_N^{\delta}(lm,\alpha) > \max\left\{A_N^{\delta}(l,\alpha), A_N^{\delta}(m,\alpha)\right\}$ or $A_P^{\delta}(lm,\alpha) < \min\left\{A_P^{\delta}(l,\alpha), A_P^{\delta}(m,\alpha)\right\}$.

If $A_N^{\delta}(lm,\alpha) > \max\left\{A_N^{\delta}(l,\alpha), A_N^{\delta}(m,\alpha)\right\}$ then $A_N^{\delta}(lm,\alpha) > s_{\alpha} \cdot \max\left\{A_N^{\delta}(l,\alpha), A_N^{\delta}(m,\alpha)\right\}$ for some $s_{\alpha} \in [-1,0]$. It follows that $lm \in N(A_{\delta}; s_{\alpha})$ but $lm \cdot N(A_{\delta}; S_{\alpha})$ which is a contradiction. Therefore $A_N^{\delta}(xy,\alpha) \cdot \max\left\{A_N^{\delta}(x,\alpha), A_N^{\delta}(y,\alpha)\right\}$ for all $x, y \in G$ and $\alpha \in \delta$. Now, if $A_P^{\delta}(lm,\alpha) < \min\left\{A_P^{\delta}(l,\alpha), A_P^{\delta}(m,\alpha)\right\}$, then $A_P^{\delta}(lm,\alpha) < t_{\alpha} \cdot \min\left\{A_P^{\delta}(l,\alpha), A_P^{\delta}(m,\alpha)\right\}$ and so $lm \in P(A_{\delta}; t_{\alpha})$ but $lm \cdot P(A_{\delta}; t_{\alpha})$. Thus $P(A_{\delta}; t_{\alpha})$ is not a soft groupoid of G, which is a contradiction. Consequently, A_{δ} is a spasmodic bipolar fuzzy soft groupoid of G.

4. Normal Spasmodic bipolar fuzzy soft groupoid

4.1 Definition : A spasmodic bipolar $\delta-$fuzzy soft groupoid $A_{\delta} = \left\{G \cdot \delta; A_P^{\delta}, A_N^{\delta}\right\}$ of G is said to be a normal if it satisfies; $A_P^{\delta}(x,\alpha) = 1$ and $A_N^{\delta}(y,\alpha) = -1$ for all $x, y \in G$.

4.2 Definition : Let $\cdot$ denote the set of all normal spasmodic bipolar fuzzy soft groupoids of G. Denote by ϕ the special element of s such that $A_P^{\delta}(\phi,\alpha) = \max_{x\in G} A_P^{\delta}(x,\alpha)$ and

$A_N^{\delta}(\phi,\alpha) = \min_{x\in G} A_N^{\delta}(x,\alpha)$ for all $\alpha \in \delta$. Clearly, if A_{δ} is a normal spasmodic bipolar $\delta-$fuzzy soft groupoid of G, then $A_P^{\delta}(\phi,\alpha) = 1$ and $A_N^{\delta}(\phi,\alpha) = -1$ for all $\alpha \in \delta$. Further we consider a method for making a normal spasmodic bipolar fuzzy soft groupoid from a given spasmodic bipolar $\delta-$fuzzy soft groupoid.

4.3 Example : Consider a group $G^{\delta} = \left\{e,a,b,c\right\}$ which is described in example 4.3.9. Let $f_{\delta} = (G^{\delta}; f_P^{\delta}, f_N^{\delta})$ be a a spasmodic bipolar fuzzy soft set in G^{δ} defined by $f_{\delta} = \left\{(e;-0.7,0.4), (a;-0.4,0.2), (b;-0.6,0.6), (e;-0.5,0.8)\right\}$. Then f_{δ} is a spasmodic bipolar fuzzy soft groupoid of G^{δ}, which induced a bipolar fuzzy soft groupoid f_{δ}, where

$A_P^{\delta}(l,1) = A_P^{\delta}(m,2) = 0.6 \simeq 1$, $A_P^{\delta}(l,2) = A_P^{\delta}(m,1) = 0.8 \simeq 1$

$A_N^{\delta}(l,1) = A_N^{\delta}(l,2) = -1, A_N^{\delta}(m,1) = A_N^{\delta}(m,2) = -1$

4.4 Theorem : Let $A_{\delta} = \left\{G \cdot \delta; A_P^{\delta}, A_N^{\delta}\right\}$ be a spasmodic bipolar $\delta-$ fuzzy soft groupoid of G. Let $\overline{A}_{\delta} = \left\{G \cdot \delta; \overline{A}_P^{\delta}, \overline{A}_N^{\delta}\right\}$ be a bipolar fuzzy soft set in G defined by $A_P^{\delta}(x,\alpha) = A_P^{\delta}(x,\alpha) - A_P^{\delta}(\phi,\alpha) + 1$ and $A_N^{\delta}(x,\alpha) = A_N^{\delta}(x,\alpha) - A_N^{\delta}(\phi,\alpha) - 1$ for all $\alpha \in \delta$ and $x \in G$. Then $\overline{A}_{\delta}$ is a normal spasmodic bipolar fuzzy soft groupoid of G.

Proof: For all $x, y \in G$ and $\alpha \in \delta$, We have

$A_P^{\delta}(xy,\alpha) \quad = A_P^{\delta}(xy,\alpha) - A_P^{\delta}(\phi,\alpha) + 1$

$$\geq \min\left\{A_P^{\delta}(x,\alpha), A_P^{\delta}(y,\alpha)\right\} - A_P^{\delta}(\phi,\alpha) + 1$$
$$= \min\left\{A_P^{\delta}(x,\alpha) - A_P^{\delta}(\phi,\alpha) + 1, A_P^{\delta}(y,\alpha) - A_P^{\delta}(\phi,\alpha) + 1\right\}$$
$$= \min\left\{A_P^{\delta}(x,\alpha), A_P^{\delta}(y,\alpha)\right\}$$

$$A_N^{\delta}(xy,\alpha) = A_N^{\delta}(xy,\alpha) - A_N^{\delta}(\phi,\alpha) - 1$$
$$\leq \max\left\{A_N^{\delta}(x,\alpha), A_N^{\delta}(y,\alpha)\right\} - A_N^{\delta}(\phi,\alpha) - 1$$
$$= \max\left\{A_N^{\delta}(x,\alpha) - A_N^{\delta}(\phi,\alpha) - 1, A_N^{\delta}(y,\alpha) - A_N^{\delta}(\phi,\alpha) - 1\right\}$$
$$= \max\left\{A_N^{\delta}(x,\alpha), A_N^{\delta}(y,\alpha)\right\}$$

$\therefore$ $\overline{A}_\delta$ is a normal spasmodic bipolar $\delta-$fuzzy soft groupoid of G.

4.5 Definition : Let $\Omega: K \to H$ be a homomorphism of groups and let $S_\delta = \left\{H \cdot \delta; S_P^{\delta}, S_N^{\delta}\right\}$ be a spasmodic bipolar fuzzy soft set in H. Then the inverse image of S_δ denoted by $\Omega^{-1}(S_\delta) = \left\{H \cdot \delta; \Omega^{-1}(S_P^{\delta}), \Omega^{-1}(S_N^{\delta})\right\}$ is the bipolar $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ fuzzy soft set in K. Given by $\Omega^{-1}(S_P^{\delta})(x,\alpha) = (S_P^{\delta})(\Omega(x),\alpha)$ and $\Omega^{-1}(S_N^{\delta})(x,\alpha) = (S_N^{\delta})(\Omega(x),\alpha)$ for all $\alpha \in \delta$ and $x \in K$ .Conversely, let A_δ be a spasmodic bipolar fuzzy soft set in K. The image of A_δ written as $\Omega(A_\delta) = \left\{H \cdot \delta; \Omega(A_P^{\delta}), \Omega(A_N^{\delta})\right\}$ is a spasmodic bipolar fuzzy soft set in H defined by

$$\Omega(A_P^{\delta})(y,\alpha) = \begin{cases} \max\limits_{z\in\Omega^{-1}(y)} A_P^{\delta}(z,\alpha) & if \quad \Omega^{-1}(y) \neq \phi \\ 0 & otherwise \end{cases}$$ and

$$\Omega(A_N^{\delta})(y,\alpha) = \begin{cases} \min\limits_{z\in\Omega^{-1}(y)} A_N^{\delta}(z,\alpha) & if \quad \Omega^{-1}(y) \neq \phi \\ 0 & otherwise \end{cases}$$ for all $\alpha \in \delta$ and $y \in H$,

where $\Omega^{-1}(y) = \left\{x / \Omega(x) = y\right\}$.

4.6 Theorem : Let $\Omega: K \to H$ be a homomorphism of groups and let S_δ be a spasmodic bipolar $\delta-$ fuzzy soft groupoid of H. Then its inverse image $\Omega^{-1}(S_\delta)$ is a spasmodic bipolar fuzzy soft groupoid of K.

Proof: Let $x, y \in K$ and $\alpha \in \delta$

$$\Omega^{-1}(S_P^{\delta})(xy,\alpha) = (S_P^{\delta})(\Omega(xy),\alpha)$$
$$= (S_P^{\delta})(\Omega(x)\Omega(y),\alpha) \qquad (\because \Omega \text{ is Homomorphism})$$
$$\geq \min\left\{S_P^{\delta}(\Omega(x),\alpha), S_P^{\delta}(\Omega(y),\alpha)\right\}$$
$$= \min\left\{\Omega^{-1}(S_P^{\delta})(x,\alpha), \Omega^{-1}(S_P^{\delta})(y,\alpha)\right\}$$ and

$$\Omega^{-1}(S_N^{\delta})(xy,\alpha) = (S_N^{\delta})(\Omega(xy),\alpha)$$
$$= (S_N^{\delta})(\Omega(x)\Omega(y),\alpha) \qquad (\because \Omega \text{ is Homomorphism})$$

$$\leq \max\left\{S_N{}^{\delta}(\Omega(x),\alpha),\ S_N{}^{\delta}(\Omega(y),\alpha)\right\}$$
$$= \max\left\{\Omega^{-1}(S_P{}^{\delta})(x,\alpha),\ \Omega^{-1}(S_N{}^{\delta})(y,\alpha)\right\}$$

Hence $\Omega^{-1}(S_\delta)$ is a spasmodic bipolar δ –fuzzy soft groupoid of K.

4.7 Theorem: Let $\Omega: K \to H$ be a homomorphism between the groups K and H. If A_δ is a spasmodic bipolar fuzzy soft groupoid of K, then the image $\Omega(A_\delta)$ is a bipolar fuzzy soft groupoid of H.

Proof: In this statement, we first show that

$\Omega^{-1}(y_1)\Omega^{-1}(y_2) \subset \Omega^{-1}(y_1 y_2)$ for all $y_1, y_2 \in H$ ⟶ (1)

For, if $x \in \Omega^{-1}(y_1)\Omega^{-1}(y_2)$ then $x = x_1 x_2$ for some $x_1 \in \Omega^{-1}(y_1)$ and $x_2 \in \Omega^{-1}(y_2)$. Since Ω is a homomorphism, it follows that $\Omega(x) = \Omega(x_1 x_2) = \Omega(x_1)\Omega(x_2) = y_1 y_2$.

So that $x \in \Omega^{-1}(y_1 y_2)$. Hence equation (1) holds

Now let $y_1, y_2 \in H$ and $\alpha \in \delta$,

Assume that $y_1, y_2 \notin I(\Omega)$, then $\Omega(A_P{}^{\delta})(y_1 y_2,\alpha) = \Omega(A_N{}^{\delta})(y_1 y_2,\alpha) = 0$ but if $y_1, y_2 \notin \mathrm{Im}(\Omega)$

,then $\Omega^{-1}(y_1) = \phi$ (or) $\Omega^{-1}(y_2) \neq \phi$ by equation (1).

Thus $\Omega(A_P{}^{\delta})(y_1,\alpha) = \Omega(A_P{}^{\delta})(y_2,\alpha) = 0$ or $\Omega(A_P{}^{\delta})(y_2,\alpha) = \Omega(A_P{}^{\delta})(y_2,\alpha) = 0$ and

$$\Omega(A_P{}^{\delta})(y_1 y_2,\alpha) = \max\left\{\Omega(A_P{}^{\delta})(y_1,\alpha), \Omega(A_P{}^{\delta})(y_2,\alpha)\right\} = 0$$
$$\Omega(A_N{}^{\delta})(y_1 y_2,\alpha) = \min\left\{\Omega(A_N{}^{\delta})(y_1,\alpha), \Omega(A_N{}^{\delta})(y_2,\alpha)\right\} = 0$$

Suppose $\Omega^{-1}(y_1 y_2) \neq \phi$, then we consider, two cases as follows

(i) $\Omega^{-1}(y_1) = \phi$ (or) $\Omega^{-1}(y_2) = \phi$

(ii) $\Omega^{-1}(y_1) = \phi$ and $\Omega^{-1}(y_2) \neq \phi$

Case(i) we have $\Omega(A_P{}^{\delta})(y_1,\alpha) = \Omega(A_N{}^{\delta})(y_1,\alpha) = 0$, $\Omega(A_P{}^{\delta})(y_2,\alpha) = \Omega(A_N{}^{\delta})(y_2,\alpha) = 0$.

Hence $\Omega(A_P{}^{\delta})(y_1 y_2,\alpha) \leq \max\left\{\Omega(A_P{}^{\delta})(y_1,\alpha), \Omega(A_P{}^{\delta})(y_2,\alpha)\right\}$ and

$\Omega(A_N{}^{\delta})(y_1 y_2,\alpha) \geq \min\left\{\Omega(A_N{}^{\delta})(y_1,\alpha), \Omega(A_N{}^{\delta})(y_2,\alpha)\right\}$

Case (ii) $\Omega(A_P{}^{\delta})(y_1 y_2,\alpha) = \max\limits_{z \in \Omega^{-1}(y_1 y_2)} A_P{}^{\delta}(z,\alpha)$

$$\leq \max_{z \in \Omega^{-1}(y_1)\Omega^{-1}(y_2)} A_P{}^{\delta}(z,\alpha)$$
$$= \max_{\substack{x_1 \in \Omega^{-1}(y_1)\\ x_2 \in \Omega^{-1}(y_2)}} A_P{}^{\delta}(x_1 x_2,\alpha)$$
$$\leq \max_{\substack{x_1 \in \Omega^{-1}(y_1)\\ x_2 \in \Omega^{-1}(y_2)}} \left\{\min\left\{A_P{}^{\delta}(x_1,\alpha), A_P{}^{\delta}(x_2,\alpha)\right\}\right\}$$

$$= \min\left\{ \max_{x_1 \in \Omega^{-1}(y_1)} A_P^{\delta}(x_1,\alpha), \max_{x_2 \in \Omega^{-1}(y_2)} A_P^{\delta}(x_2,\alpha) \right\}$$

$$= \min\left\{ \Omega(A_P^{\delta})(y_1,\alpha), \Omega(A_P^{\delta})(y_2,\alpha) \right\}$$ and

$$\Omega(A_N^{\delta})(y_1 y_2,\alpha) = \min_{z \in \Omega^{-1}(y_1 y_2)} A_N^{\delta}(z,\alpha)$$

$$\leq \min_{Z \in \Omega^{-1}(y_1)\Omega^{-1}(y_2)} A_N^{\delta}(z,\alpha)$$

$$= \min_{\substack{x_1 \in \Omega^{-1}(y_1) \\ x_2 \in \Omega^{-1}(y_2)}} A_N^{\delta}(x_1 x_2,\alpha)$$

$$\leq \min_{\substack{x \in \Omega^{-1}(y_1) \\ x \in \Omega^{-1}(y_2)}} \left\{ \max\left\{ A_N^{\delta}(x_1,\alpha), A_N^{\delta}(x_2,\alpha) \right\} \right\}$$

$$= \max\left\{ \min_{x_1 \in \Omega^{-1}(y_1)} A_N^{\delta}(x_1,\alpha), \min_{x_2 \in \Omega^{-1}(y_2)} A_N^{\delta}(x_2,\alpha) \right\}$$

$$= \max\left\{ \Omega(A_N^{\delta})(y_1,\alpha), \Omega(A_N^{\delta})(y_2,\alpha) \right\}$$ for all $y_1, y_2 \in H$ and $\alpha \in \delta$.

5. Construction of fuzzy soft translation

In this section we will discuss the basic idea of soft translation.

5.1 Definition : Let $A_{\delta} = (G \times \delta; A_P^{\delta}, A_N^{\delta})$ be a spasmodic bipolar fuzzy soft set of G and $(s,t) \in [-1,0] \times [0,1]$. By a bipolar fuzzy soft (s, t) translation of A_{δ}. We mean a bipolar $\delta-$fuzzy soft set $A_{\delta}^{(s,t)} = (G \times \delta; A_P^{\delta(t,T)}, A_N^{\delta(s,T)})$ where

$A_{P^{\delta}}^{(t,T)} : G \times \delta \to [0,1]$ is a mapping defined by $A_{P^{\delta}}^{(t,T)}(x) = A_P^{\delta}(x,\alpha) + t \;\forall\; x \in G, \alpha \in G$ and

$A_{N^{\delta}}^{(s,T)} : G \times \delta \to [-1,0]$ is a mapping defined by $A_{N^{\delta}}^{(s,T)}(x) = A_N^{\delta}(x,\alpha) + s \;\forall\; x \in G, \alpha \in G$.

5.2 Definition : A spasmodic bipolar fuzzy soft groupoid A_{δ} of a groupoid of G is called a bipolar $\delta-$fuzzy soft bi-ideal of G if

(i) $A_P^{\delta}(xyz) \geq \min\left\{ A_P^{\delta}(x), A_P^{\delta}(z) \right\}$

(ii) $A_N^{\delta}(xyz) \leq \max\left\{ A_N^{\delta}(x), A_N^{\delta}(z) \right\}$ for all $x, y, z \in G$.

5.3 Example : In example 3.9 we defined spasmodic bipolar fuzzy soft set $A_{\delta} = (G \times \delta; A_P^{\delta}, A_N^{\delta})$ of G as follows

G^{δ}	e	a	b	c
A_P^{δ}	0.6	0.2	0.5	0.3
A_N^{δ}	-0.7	-0.4	-0.9	-0.9

Let $s = -0.1$ and t=0.4.Then the bipolar fuzzy soft (s, t) translation $A_{(s,t)}^{\delta T} = (G \cdot \delta ; A_P^{\delta}(s,T), A_N^{\delta}(t,T))$ of $A_{\delta} = (G \cdot \delta ; A_P^{\delta}, A_N^{\delta})$ is

G^{δ}	e	a	b	c
$A_P^{\delta}(s,T)$	0.9	0.6	0.9	0.7
$A_N^{\delta}(s,T)$	-0.8	-0.5	-1.0	-1.0

5.4 Theorem : Let $A_{\delta} = (G \cdot \delta ; A_P^{\delta}, A_N^{\delta})$ be a non-empty bipolar fuzzy soft subset of G and $(s,t) \in [-1,0] \cdot [0,1]$. Then the spasmodic bipolar fuzzy soft translation A_{δ}^{T} is a spasmodic bipolar fuzzy soft groupoid of G if and only if A_{δ} is a bipolar fuzzy soft groupoid of G.

Proof: Let A_{δ} be a spasmodic bipolar fuzzy soft groupoid of G and $x, y \in G^{\delta}$. Then

$$A_{P^{\delta}}^{(t,T)}(xy) = A_P^{\delta}(xy,\alpha) + t$$

$$\cdot \min\left\{A_P^{\delta}(x,\alpha), A_P^{\delta}(y,\alpha)\right\} + t$$

$$= \min\left\{A_P^{\delta}(x,\alpha) + t, A_P^{\delta}(y,\alpha) + t\right\}$$

$$= \min\left\{A_{P^{\delta}}^{(t,T)}(x), A_{P^{\delta}}^{(t,T)}(y)\right\}$$

$$A_{N^{\delta}}^{(s,T)}(xy) = A_N^{\delta}(xy,\alpha) + s$$

$$\cdot \max\left\{A_N^{\delta}(x,\alpha), A_N^{\delta}(y,\alpha)\right\} + s$$

$$= \max\left\{A_N^{\delta}(x,\alpha) + s, A_N^{\delta}(y,\alpha) + s\right\}$$

$$= \max\left\{A_{N^{\delta}}^{(s,T)}(x), A_{N^{\delta}}^{(s,T)}(y)\right\}$$

Hence $A_{\delta}^{T}(s,t)$ is a spasmodic bipolar fuzzy soft groupoid of G.

Conversely, let $A_{\delta}^{T}(s,t)$ be a spasmodic bipolar fuzzy soft groupoid of G for some $(s,t) \in [-1,0] \cdot [0,1]$. Then for any $x, y \in G^{\delta}$ we have

$$A_P^{\delta}(xy,\alpha) + t = A_{P^{\delta}}^{(t,T)}(xy)$$

$$\cdot \min\left\{A_{P^{\delta}}^{(t,T)}(x), A_{P^{\delta}}^{(t,T)}(y)\right\}$$

$$= \min\left\{A_P^{\delta}(x,\alpha) + t, A_P^{\delta}(y,\alpha) + t\right\}$$

$$= \min\left\{A_P^{\delta}(x), A_P^{\delta}(y)\right\} \text{and}$$

$$A_N^{\delta}(xy,\alpha) + s = A_{N}^{\delta\,(s,T)}(xy,\alpha)$$

$$\cdot \max\left\{A_{N^{\delta}}^{(s,T)}(x), A_{N^{\delta}}^{(s,T)}(y)\right\}$$

$$= \max\left\{A_N^{\delta}(x,\alpha)+s, A_N^{\delta}(y,\alpha)+s\right\}$$
$$= \max\left\{A_N^{\delta}(x), A_N^{\delta}(y)\right\}$$

which implies that A_δ is a spasmodic bipolar fuzzy soft groupoid of G.

5.5 Definition : Let $A_\delta = (G \cdot \delta; A_P^{\delta}, A_N^{\delta})$ and $A_\cdot = (G \cdot \cdot; A_P^{\cdot}, A_N^{\cdot})$ be two bipolar $\delta-$fuzzy soft sets of G^δ. If $A_P^{\delta}(x,\alpha) \cdot A_P^{\cdot}(x,\alpha)$ and $A_N^{\delta}(x,\alpha) \cdot A_N^{\cdot}(x,\alpha)$ for all $x \in G^\delta$ and $\alpha \in \delta$. Then we say that $A_\cdot$ is spasmodic bipolar fuzzy soft extension of A_δ.

5.6 Example : Let $A_\cdot = (G \cdot \cdot; A_P^{\cdot}, A_N^{\cdot})$ be a spasmodic bipolar fuzzy soft set of a groupoid of G^δ in example 4.5.3 and it is define as follows,

G^δ	e	a	b	c
$A_P^{\cdot}$	0.63	0.62	0.72	0.75
$A_N^{\cdot}$	-0.83	-0.52	-0.92	-0.92

Then $A_\cdot$ is a spasmodic bipolar fuzzy soft extension of A_δ.

6. Spasmodic Bipolar fuzzy s-extension of soft sub groupoid

6.1 Definition : let $A_\cdot$ and A_δ be two bipolar fuzzy soft sets. Then $A_\cdot$ is called Spacemodic bipolar fuzzy $s-$extension of A_δ if the following hold

(i) $A_\cdot$ is spasmodic bipolar $\delta-$fuzzy extension of A_δ.

(ii) A_δ is a spasmodic bipolar $\delta-$fuzzy soft groupoid of G^δ.

6.2 Definition : The union of any two spasmodic bipolar $\delta-$fuzzy soft sets $A_\cdot$ and A_δ is a bipolar $\delta-$fuzzy soft set $(A_\delta \cup A_\cdot)^\delta(x,\alpha) = \min\left\{A_\delta(x,\alpha), A_\cdot(x,\alpha)\right\}$ where $(A_\delta \cup A_\cdot)^\delta : G \cdot \delta \to [0,1]$ and $(A_\delta \cap A_\cdot)(x,\alpha) = \max\left\{A_\delta(x,\alpha), A_\cdot(x,\alpha)\right\}$ where $(A_\delta \cap A_\cdot)^\delta : G \cdot \delta \to [-1,0]$ for all $x \in G^\delta$ and $\alpha \in \delta$.

6.3 Theorem: Union of two bipolar fuzzy soft $s-$extension of a spasmodic bipolar fuzzy soft groupoid A_δ in G^δ is a spasmodic bipolar fuzzy soft $s-$extension of A_δ in G^δ.

Proof :

Let $A_\cdot = (G \cdot \delta; A_P^{\cdot}, A_N^{\cdot})$ and $A_\pi = (G \cdot \delta; A_P^{\pi}, A_N^{\pi})$ be two spasmodic bipolar $\delta-$fuzzy soft $s-$ extension of A_δ in G^δ. Then $A_P^{\delta}(x,\alpha) \cdot A_P^{\cdot}(x,\alpha)$, $A_P^{\delta}(x,\alpha) \cdot A_P^{\pi}(x,\alpha)$ and $A_N^{\delta}(x,\alpha) \cdot A_N^{\cdot}(x,\alpha)$, $A_N^{\delta}(x,\alpha) \cdot A_N^{\pi}(x,\alpha)$ for all $x \in G^\delta$ and $\alpha \in \delta$. Now

$(A_{\cdot} \cup A_{\pi})_P(x,\alpha) = \max\{A_P^{\cdot}(x,\alpha), A_P^{\pi}(x,\alpha)\} \cdot A_P^{\delta}(x,\alpha)$ and

$(A_{\cdot} \cup A_{\pi})_N(x,\alpha) = \min\{A_N^{\cdot}(x,\alpha), A_N^{\pi}(x,\alpha)\} \cdot A_N^{\delta}(x,\alpha)$

Consequently, let $A_{\cdot} \cup A_{\pi} = (G \cdot \delta; (A_{\cdot} \cup A_{\pi})_{P^{\delta}}, (A_{\cdot} \cup A_{\pi})_{N^{\delta}})$ is a spasmodic bipolar fuzzy soft $s-$extension of A_{δ}. Since A_{δ} is a spasmodic bipolar fuzzy soft groupoid of G^{δ}, So $A_{\cdot} \cup A_{\pi}$ is a spasmodic bipolar $\delta-$fuzzy soft $s-$extension of A_{δ}.

6.4 Theorem : Intersection of two spasmodic bipolar fuzzy soft $s-$extension of a spasmodic bipolar $\delta-$fuzzy soft groupoid A_{δ} in G^{δ} is a spasmodic bipolar fuzzy soft $s-$extension of A_{δ} in G^{δ}.

Proof: Let $A_{\cdot} = (G \cdot \delta; A_P^{\cdot}, A_N^{\cdot})$ and $A_{\pi} = (G \cdot \delta; A_P^{\pi}, A_N^{\pi})$ be two spasmodic bipolar fuzzy soft $s-$extension of A_{δ} in G^{δ}. Then

$A_P^{\delta}(x,\alpha) \cdot A_P^{\cdot}(x,\alpha)$, $A_P^{\delta}(x,\alpha) \cdot A_P^{\pi}(x,\alpha)$, and

$A_N^{\delta}(x,\alpha) \cdot A_N^{\cdot}(x,\alpha)$, $A_N^{\cdot}(x,\alpha) \cdot A_N^{\pi}(x,\alpha)$ for all $x \in G^{\delta}$ and $\alpha \in \delta$.

Now

$(A_{\cdot} \cap A_{\pi})_P(x,\alpha) = \min\{A_P^{\cdot}(x,\alpha), A_P^{\pi}(x,\alpha)\} \cdot A_P^{\delta}(x,\alpha)$ and

$(A_{\cdot} \cap A_{\pi})_N(x,\alpha) = \max\{A_N^{\cdot}(x,\alpha), A_N^{\pi}(x,\alpha)\} \cdot A_N^{\delta}(x,\alpha)$

Consequently, let $A_{\cdot} \cap A_{\pi} = (G \cdot \delta; (A_{\cdot} \cap A_{\pi})_{P^{\delta}}, (A_{\cdot} \cap A_{\pi})_{N^{\delta}})$ is a bipolar $\delta-$fuzzy soft $s-$extension of A_{δ}. Since A_{δ} is a spasmodic bipolar fuzzy soft groupoid of G^{δ}, So $A_{\cdot} \cap A_{\pi}$ is a spasmodic bipolar $\delta-$fuzzy soft $s-$extension of A_{δ}.

6.5 Example : $A_{\cdot}$ is example 4.5.6 in a spasmodic bipolar fuzzy soft $s-$extension of A_{δ} in example (4.5 6). Let $A_{\delta} = (G \cdot \delta; A_P^{\delta}, A_N^{\delta})$ be $BP\delta FSS$. Then for all $x, y \in G^{\delta}$ and $\alpha \in \delta$, We have spasmodic bipolar fuzzy soft (s,t)- translation is defined by $A_{P^{\delta}}^{(t,T)}(x,\alpha) \cdot A_P^{\delta}(x,\alpha)$ and $A_{N^{\delta}}^{(s,T)}(x,\alpha) \cdot A_N^{\delta}(x,\alpha)$ all $x, y \in G^{\delta}$ and $\alpha \in \delta$.

6.6 Remark: Let A_{δ} be a spasmodic bipolar fuzzy soft groupoid of G^{δ} and $(s,t) \in [-1,0] \cdot [0,1]$. Then spasmodic bipolar fuzzy soft (s, t)-translation $A^T_{(s,T)}$ is a spasmodic bipolar $\delta-$fuzzy $s-$extension of A_{δ}. The converse of the above theorem is not in general. In fact $A_{\cdot}$ in example (4.6.5) is a bipolar A_{δ} in G^{δ}.

7. Decision making approach for spasmodicbipolar fuzzy soft set

Algorithm:

Step(1) Construct D_1complement and D_2 complement for the decision matrix

Step(2) Calculate $BF\delta(D_1^C, D_2^C)$ and $BF\delta(D_2^C, D_1^C)$

Step (3) Calculate score function

Step (4) Calculate the correlation coefficient between diseases and symptoms

Step (5) Choose maximum value

Step (6) Choose rank the order

Numerical example we construct the following bipolar value problem based on the above algorithm

Spasmodic Bipolar fuzzy soft set D_1

D_1 =

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[-0.2, 0.5]	[-0.1, 0.6]	[-0.3, 0.7]	[-0.2, 0.6]
α_2	[-0.1, 0.4]	[-0.2, 0.5]	[-0.2, 0.6]	[-0.1, 0.5]
α_3	[-0.3, 0.4]	[-0.1, 0.4]	[-0.1, 0.5]	[-0.1, 0.4]

Spasmodic Bipolar fuzzy soft set D_2

D_2 =

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[-0.1, 0.3]	[-0.3, 0.4]	[-0.1, 0.3]	[-0.2, 0.5]
α_2	[-0.2, 0.4]	[-0.1, 0.5]	[-0.3, 0.4]	[-0.1, 0.4]
α_3	[-0.3, 0.5]	[-0.2, 0.3]	[-0.1, 0.2]	[-0.3, 0.2]

Spasmodic Bipolar fuzzy soft set D_1 Complement

$D_1{}^C$ =

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[-0.8, 0.5]	[-0.9, 0.4]	[-0.7, 0.3]	[-0.8, 0.4]
α_2	[-0.9, 0.6]	[-0.8, 0.5]	[-0.8, 0.4]	[-0.9, 0.5]
α_3	[-0.7, 0.6]	[-0.9, 0.6]	[-0.9, 0.5]	[-0.9, 0.6]

Spasmodic Bipolar fuzzy soft set D_2 Complement

$D_2{}^C$ =

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[-0.9, 0.7]	[-0.7, 0.6]	[-0.9, 0.7]	[-0.8, 0.5]
α_2	[-0.8, 0.6]	[-0.9, 0.5]	[-0.7, 0.6]	[-0.9, 0.6]
α_3	[-0.7, 0.5]	[-0.8, 0.7]	[-0.9, 0.8]	[-0.7, 0.8]

Step (2) Calculate $BF\delta(D_1{}^C, D_2{}^C)$

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
$\alpha_1\ \alpha_2$	[-0.8, 0.5]	[-0.8, 0.4]	[-0.7, 0.3]	[-0.8, 0.4]
$\alpha_2\ \alpha_3$	[-0.7, 0.6]	[-0.8, 0.5]	[-0.8, 0.4]	[-0.9, 0.5]
$\alpha_1\ \alpha_3$	[-0.7, 0.5]	[-0.9, 0.4]	[-0.7, 0.3]	[-0.8, 0.4]

Calculate $BF\delta(D_2^C, D_1^C)$

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[-0.8, 0.7]	[-0.7, 0.5]	[-0.7, 0.6]	[-0.8, 0.5]
α_2	[-0.7, 0.5]	[-0.8, 0.5]	[-0.7, 0.6]	[-0.7, 0.6]
α_3	[-0.7, 0.5]	[-0.7, 0.6]	[-0.9, 0.7]	[-0.8, 0.5]

Step (3): Calculate score function $A_P^\delta = \sum_{i=1}^{n} \left(\frac{a+b}{3} \right)$ and $A_N^\delta = \sum_{j=1}^{n} \left(\frac{a+b}{3} \right)$

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[-0.533, 0.400]	[-0.530, 0.330]	[-0.466, 0.300]	[-0.330, 0.300]
α_2	[-0.466, 0.366]	[-0.533 0.330]	[-0.500, 0.330]	[-0.530, 0.360]
α_3	[-0.466, 0.333]	[-0.533, 0.330]	[-0.533, 0.330]	[-0.533, 0.330]

Score function $S = \frac{c+d}{2} = \frac{-5.959+4.012}{2} = -0.9735$

Where c is the sum of negative value and d is the sum of positive value.

Step (4): Calculate correlation coefficient between diseases and symptoms by using the formula is given by

$$p = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}(d_{ij})^2}{\sqrt{\sum_{i=1}^{n}(d_i)^2}\sqrt{\sum_{j=1}^{n}(d_j)^2}}$$

Using step(3) we form a new calculation table as follows

Y \ X	1	2	3	4
α_1	-0.13	-0.230	-0.166	-0.030

α_2	-0.10	-0.203	-0.170	-0.170
α_3	-0.13	-0.203	-0.203	-0.203

$$p = 0.3108 < 1$$

Step (5): Score function = -0.9735 = -0.97

Correlation function = 0.31

mMaximum value =0.31

Step (6): From the table the order preference is given from step (4) as given below

Rank the order $r_4 < r_1 < r_3 < r_2$

$r_1 < r_3 < r_4 < r_2$

$r_1 < r_2 < r_3 < r_4$

For α_1 symptoms Cancer <Fever <Typhoid <Headache

For α_2 symptoms Fever<Typhoid <Cancer<Headache

For α_3 symptoms Fever <Headache < Typhoid<Cancer

Result: Finally we conclude that symptoms (3)(ie) α_3 which is decide to choose the nearest ranking order for the given problem.

Conclusion: The construction of fuzzy soft translation has been explained. It has been stated, how the homomorphic images and the inverse images of spasmodic bipolar fuzzy soft groupoid are used. The s- extension of the soft set groupoid from the spasmodic bipolar fuzzy set has been described, using spasmodic bipolar fuzzy soft groupoid. In the bipolar fuzzy soft set, the decision making approach has been applied

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