

BIPOLAR INTERVAL VALUED INTUITIONISTIC FUZZY NECESSITY OPERATOR

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Abstract

In this paper we have introduced the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified its property.

Keyword

Bipolar Interval Valued Intuitionistic Fuzzy Topological Space, Bipolar Interval Valued Intuitionistic Fuzzy Set.

1. Introduction:

Lee introduced the concept of Bipolar fuzzy set. In Bipolar Intuitionistic Fuzzy Topology the membership and non-membership degree of the fuzzy set lies in the range [0,1] and [-1,0][21]. In this paper we have introduced the Bipolar Interval Valued Intuitionistic Fuzzy necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified that the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topological space.

2. Definition:

Let X be a non-empty set, and let A be a Bipolar interval valued intuitionistic fuzzy set on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X), then the necessity operator on A is defined as

$$\text{i. } \quad []A = \left\{ \left\langle x, \begin{bmatrix} \underline{\mathbb{P}}^P(x), \underline{\mathbb{P}}^P(x) \\ \underline{\mathbb{P}}^{AL}(x), \underline{\mathbb{P}}^{AU}(x) \end{bmatrix}, \begin{bmatrix} 1 - \underline{\mathbb{P}}^P(x), 1 - \underline{\mathbb{P}}^P(x) \\ \square 1 + \underline{\mathbb{P}}^{AU}(x), \square 1 + \underline{\mathbb{P}}^{AL}(x) \end{bmatrix} \right\rangle \mid x \in X \right\}$$

2.1. Theorem:

Let (X, \mathbb{I}) be a Bipolar Interval Valued Intuitionistic Fuzzy Topological Space (BIVIFTS). Based on the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy set A on X, we can also construct several BIVIFTSs on X as

$$\mathbb{Q}_N = \{ []A \mid A \sqsubseteq \mathbb{Q} \}$$

i.e., the necessity operator defined in the above definition itself forms a topology.

Proof:

In order to prove the topology we have to prove the following

Let S be a set and \mathcal{B} be a family of bipolar interval valued intuitionistic fuzzy subset of S. The family is called a Bipolar Interval Valued Intuitionistic Fuzzy Topology (BIVIFT) on S if satisfies the following axioms

i. $0_s, 1_s \sqsubseteq \mathbb{Q}$

ii. If $\{A_i; i \in I\} \sqsubseteq \mathbb{Q}$, then $\bigcup_{i=1}^{\square} A_i \sqsubseteq \mathbb{Q}$

iii. If $A_1, A_2, A_3 \dots A_n \sqsubseteq \mathbb{Q}$, then $\bigcup_{i=1}^n A_i \sqsubseteq \mathbb{Q}$

Let A_1, A_2, \dots, A_i be Bipolar interval valued intuitionistic fuzzy subsets on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X).

To prove necessity operator is a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X)

i. obviously $0_s, 1_s \sqsubseteq \mathbb{Q}_N$

ii.

$$A \sqsubseteq B = \left\langle \begin{array}{l} \left[x, \mathbb{Q}_{(A \sqsubseteq B)L} (x), \mathbb{Q}_{(A \sqsubseteq B)U}^p (x) \right], \left[\mathbb{Q}_{(A \sqsubseteq B)L}^N (x), \mathbb{Q}_{(A \sqsubseteq B)U}^N (x) \right], \\ \left[\mathbb{Q}_{(A \sqsubseteq B)L}^p (x), \mathbb{Q}_{(A \sqsubseteq B)U}^p (x) \right], \left[\mathbb{Q}_{(A \sqsubseteq B)L}^N (x), \mathbb{Q}_{(A \sqsubseteq B)U}^N (x) \right] \end{array} \right\rangle \mid x \in X$$

where

$$\mathbb{Q}_{(A \sqsubseteq B)L}^p (x) = \min \left\{ \mathbb{Q}_{AL}^p (x), \mathbb{Q}_{BL}^p (x) \right\}$$

$$\begin{aligned}\hat{\mathbb{E}}_{(A \square B)U}^p(x) &= \max \left\{ \hat{\mathbb{E}}_{AU}^p(x), \hat{\mathbb{E}}_{BU}^p(x) \right\} \\ \hat{\mathbb{E}}_{(A \square B)L}^N(x) &= \max \left\{ \hat{\mathbb{E}}_{AL}^N(x), \hat{\mathbb{E}}_{BL}^N(x) \right\}\end{aligned}$$

$$\hat{\mathbb{E}}_{(A \square B)U}^N(x) = \min \left\{ \hat{\mathbb{E}}_{AU}^N(x), \hat{\mathbb{E}}_{BU}^N(x) \right\}$$

$$\hat{\mathbb{E}}_{(A \square B)L}^p(x) = \min \left\{ \hat{\mathbb{E}}_{AL}^p(x), \hat{\mathbb{E}}_{BL}^p(x) \right\}$$

$$\begin{aligned}\hat{\mathbb{E}}_{(A \square B)U}^p(x) &= \max \left\{ \hat{\mathbb{E}}_{AU}^p(x), \hat{\mathbb{E}}_{BU}^p(x) \right\} \\ \hat{\mathbb{E}}_{(A \square B)L}^N(x) &= \max \left\{ \hat{\mathbb{E}}_{AL}^N(x), \hat{\mathbb{E}}_{BL}^N(x) \right\}\end{aligned}$$

$$\hat{\mathbb{E}}_{(A \square B)U}^N(x) = \min \left\{ \hat{\mathbb{E}}_{AU}^N(x), \hat{\mathbb{E}}_{BU}^N(x) \right\}$$

$$\boxed{\boxed{[A_1 \quad A_2]}} = \boxed{\boxed{\begin{array}{c} \boxed{x, \hat{\mathbb{E}}_{[A_1 \square [A_2]L}(x), \hat{\mathbb{E}}_{[A_1 \square [A_2]U}(x)} \\ \boxed{\hat{\mathbb{E}}_{[A_1 \square [A_2]L}(x), \hat{\mathbb{E}}_{[A_1 \square [A_2]U}(x)} \\ \boxed{\hat{\mathbb{E}}_{[A_1 \square [A_2]L}^p(x), \hat{\mathbb{E}}_{[A_1 \square [A_2]U}^p(x)} \\ \boxed{\hat{\mathbb{E}}_{[A_1 \square [A_2]L}^N(x), \hat{\mathbb{E}}_{[A_1 \square [A_2]U}^N(x)} \end{array}}} \boxed{| x \square X}}$$

where

$$\hat{\mathbb{E}}_{([A_1 \square [A_2]L}^p(x) = \min \left\{ \hat{\mathbb{E}}_{[A_1 L}^p(x), \hat{\mathbb{E}}_{[A_2 L}^p(x) \right\}$$

$$\hat{\mathbb{E}}_{([A_1 \square [A_2]U}^p(x) = \max \left\{ \hat{\mathbb{E}}_{[A_1 U}^p(x), \hat{\mathbb{E}}_{[A_2 U}^p(x) \right\}$$

$$\hat{\mathbb{E}}_{([A_1 \square [A_2]L}^N(x) = \max \left\{ \hat{\mathbb{E}}_{[A_1 L}^N(x), \hat{\mathbb{E}}_{[A_2 L}^N(x) \right\}$$

$$\hat{\mathbb{E}}_{([A_1 \square [A_2]U}^N(x) = \min \left\{ \hat{\mathbb{E}}_{[A_1 U}^N(x), \hat{\mathbb{E}}_{[A_2 U}^N(x) \right\}$$

$$\hat{\mathbb{E}}_{([A_1 \square [A_2]L}^p(x) = \min \left\{ \hat{\mathbb{E}}_{[A_1 L}^p(x), \hat{\mathbb{E}}_{[A_2 L}^p(x) \right\}$$

$$\hat{\mathbb{E}}_{([A_1 \square [A_2]U}^p(x) = \max \left\{ \hat{\mathbb{E}}_{[A_1 U}^p(x), \hat{\mathbb{E}}_{[A_2 U}^p(x) \right\}$$

$$\hat{\mathbb{E}}_{([A_1 \square [A_2]L}^N(x) = \max \left\{ \hat{\mathbb{E}}_{[A_1 L}^N(x), \hat{\mathbb{E}}_{[A_2 L}^N(x) \right\}$$

$$\hat{\mathbb{E}}_{([A_1 \square [A_2]U}^N(x) = \min \left\{ \hat{\mathbb{E}}_{[A_1 U}^N(x), \hat{\mathbb{E}}_{[A_2 U}^N(x) \right\}$$

then

$$\hat{\mathbb{E}}_{([A_1 \square [A_2]L}^p(x) = \min \left\{ \hat{\mathbb{E}}_{[A_1 L}^p(x), \hat{\mathbb{E}}_{[A_2 L}^p(x) \right\}$$

$$\hat{\mathbb{E}}_{([A_1 \square [A_2]U}^p(x) = \max \left\{ \hat{\mathbb{E}}_{[A_1 U}^p(x), \hat{\mathbb{E}}_{[A_2 U}^p(x) \right\}$$

$$\hat{\mathbb{E}}_{([A_1 \square [A_2]L}^N(x) = \max \left\{ \hat{\mathbb{E}}_{[A_1 L}^N(x), \hat{\mathbb{E}}_{[A_2 L}^N(x) \right\}$$

$$\min\left\{\mathbb{E}_{A_1U}^N(x), \mathbb{E}_{A_2U}^N(x)\right\}$$

$$\begin{aligned} 1 \square \hat{\square}^p ([\]_{A_1} [\]_{A_2})_L(x) &= \min \left\{ 1 \square \hat{\square}^p_{A_1 L}(x), 1 \square \hat{\square}^p_{A_2 L}(x) \right\} \\ 1 \square \hat{\square}^p ([\]_{A_1} [\]_{A_2})_U(x) &= \max \left\{ 1 \square \hat{\square}^p_{A_1 U}(x), 1 \square \hat{\square}^p_{A_2 U}(x) \right\} \end{aligned}$$

$$1 \square \lceil \frac{N}{\lfloor \lceil A_1 \square \lfloor \lceil A_2 \rceil \rfloor \rceil} \rceil_L(x) = \max \left\{ 1 \square \lceil \frac{N}{\lfloor \lceil A_1 \rceil \rfloor} \rceil_{A_1 L}(x), 1 \square \lceil \frac{N}{\lfloor \lceil A_2 \rceil \rfloor} \rceil_{A_2 L}(x) \right\}$$

$$1 \square \lceil \frac{N}{\lfloor \cdot \rfloor_{A_1} \lceil \cdot \rceil_{A_2}} \rfloor_U(x) = \min \left\{ 1 \square \lceil \frac{N}{\lceil \cdot \rceil_{A_1 U}}(x), 1 \square \lceil \frac{N}{\lceil \cdot \rceil_{A_2 U}}(x) \right\}$$

$$\square []A_1 \square []A_2 = \begin{cases} \left\langle x, \emptyset \right\rangle^p_{[]A_1 \square []A_2 L} ()x, \emptyset^p_{[]A_1 \square []A_2 U} (x) \Big], & x \in X \\ \left\langle \left[\emptyset^N_{[]A_1 \square []A_2 L} ()x, \emptyset^N_{[]A_1 \square []A_2 U} (x) \right], (x) \Big], & x \in X \\ \left[\left([]A_1 \square []A_2 \right)_L ()x, []A_1 \square []A_2 \right)_U (x) \Big], & x \in X \\ \left[1 \square \emptyset^p_{[]A \square []A L} ()x, 1 \square \emptyset^p_{[]A \square []A U} (x) \Big], & x \in X \end{cases}$$

where

$$\mathbb{E}^p_{\left(\begin{array}{c} \square \\ [] A_1 \square [] A_2 \square \dots [] A_i \end{array}\right) L}(x) = \min \left\{ \mathbb{E}^p_{[] A_1 L}(x), \mathbb{E}^p_{[] A_2 L}(x), \dots, \mathbb{E}^p_{[] A_i L}(x) \right\}$$

$$\boxed{2}^p (\]A_1 \square [\]A_2 \square \dots [\]A_i) U = \max \left\{ \boxed{2}^p (\]A_1 U), \boxed{2}^p (\]A_2 U), \dots, \boxed{2}^p (\]A_i U) \right\}$$

$$\hat{\mathbb{B}}^N([A_1 \square [A_2 \square \dots [A_L]_L]) (x) = \max \left\{ \hat{\mathbb{B}}^N([A_1 L] (x), \hat{\mathbb{B}}^N([A_2 L] (x), \dots, \hat{\mathbb{B}}^N([A_L L] (x) \right\}$$

$$\min \left\{ \left[\begin{array}{c} x \\ \vdots \\ x_{A_1} \end{array} \right]_{A_2 \square \dots \square A_U}(x), \left[\begin{array}{c} x \\ \vdots \\ x_{A_2} \end{array} \right]_{A_3 \square \dots \square A_U}(x), \dots, \left[\begin{array}{c} x \\ \vdots \\ x_{A_U} \end{array} \right]_{A_1 \square \dots \square A_{U-1}}(x) \right\}$$

$$\mathbb{E}^p_{\left(\begin{array}{c} \cdot \\ A_1 \end{array}\right) A_2 \square \cdots \left(\begin{array}{c} \cdot \\ A_i \end{array}\right) L}(x) = \min \left\{ \mathbb{E}^p_{\left[\begin{array}{c} \cdot \\ A_1 L \end{array}\right]}(x), \mathbb{E}^p_{\left[\begin{array}{c} \cdot \\ A_2 L \end{array}\right]}(x), \dots, \mathbb{E}^p_{\left[\begin{array}{c} \cdot \\ A_i L \end{array}\right]}(x) \right\}$$

$$\max_{\left[\begin{smallmatrix} p \\ A_1 \square A_2 \square \dots \square A_U \end{smallmatrix} \right] U} (x) = \max \left\{ \left[\begin{smallmatrix} p \\ A_1 U \end{smallmatrix} \right] \left(x \right), \left[\begin{smallmatrix} p \\ A_2 U \end{smallmatrix} \right] (x), \dots, \left[\begin{smallmatrix} p \\ A_U U \end{smallmatrix} \right] (x) \right\}$$

$$\max_{\{(\cdot)_{A_1}, \square_A, \Box_A, \ldots, (\cdot)_{A_L}\}_L} (x) = \max \left\{ \max_{\{\cdot\}_{A_1 L}} (x), \max_{\{\Box_A\}_{A_1 L}} (x), \ldots, \max_{\{\square_A\}_{A_1 L}} (x) \right\}$$

$$\min_{\{x\}_{A,U}}(x) = \min\left\{\{x\}_{A,U}(x), \dots, \{x\}_{A,U}(x)\right\}$$

then

$$\min\left\{\hat{A}_{A_1L}^p(x), \hat{A}_{A_2L}^p(x), \dots, \hat{A}_{A_LL}^p(x)\right\}$$

$$\begin{aligned}
 \exists^P_{([A_1 \square []A_2 \square \dots []A_i]_U)}(x) &= \max \left\{ \exists^P_{A_1 U}(x), \exists^P_{A_2 U}(x), \dots, \exists^P_{A_i U}(x) \right\} \\
 \exists^N_{([]A_1 \square []A_2 \square \dots []A_i)_L}(x) &= \max \left\{ \exists^N_{A_1 L}(x), \exists^N_{A_2 L}(x), \dots, \exists^N_{A_i L}(x) \right\} \\
 \exists^N_{([]A_1 \square []A_2 \square \dots []A_i)_U}(x) &= \min \left\{ \exists^N_{A_1 U}(x), \exists^N_{A_2 U}(x), \dots, \exists^N_{A_i U}(x) \right\} \\
 1 \square \exists^P_{([]A_1 \square []A_2 \square \dots []A_i)_L}(x) &= \min \left\{ 1 \square \exists^P_{A_1 L}(x), 1 \square \exists^P_{A_2 L}(x), \dots, 1 \square \exists^P_{A_i L}(x) \right\} \\
 1 \square \exists^P_{([]A_1 \square []A_2 \square \dots []A_i)_U}(x) &= \max \left\{ 1 \square \exists^P_{A_1 U}(x), 1 \square \exists^P_{A_2 U}(x), \dots, 1 \square \exists^P_{A_i U}(x) \right\} \\
 1 \square \exists^N_{([]A_1 \square []A_2 \square \dots []A_i)_L}(x) &= \max \left\{ 1 \square \exists^N_{A_1 L}(x), 1 \square \exists^N_{A_2 L}(x), \dots, 1 \square \exists^N_{A_i L}(x) \right\} \\
 1 \square \exists^N_{([]A_1 \square []A_2 \square \dots []A_i)_U}(x) &= \min \left\{ 1 \square \exists^N_{A_1 U}(x), 1 \square \exists^N_{A_2 U}(x), \dots, 1 \square \exists^N_{A_i U}(x) \right\} \\
 \exists^P_{[]A_1 \square []A_2 \square \dots []A_i} &= \exists^P_{\exists^P_{A_1 L}(x), \exists^P_{A_2 L}(x), \dots, \exists^P_{A_i L}(x)}, \quad \exists^P_{\exists^N_{A_1 U}(x), \exists^N_{A_2 U}(x), \dots, \exists^N_{A_i U}(x)} \\
 &\quad \exists^P_{\exists^N_{A_1 L}(x), \exists^N_{A_2 L}(x), \dots, \exists^N_{A_i L}(x)}, \quad \exists^P_{\exists^P_{A_1 U}(x), \exists^P_{A_2 U}(x), \dots, \exists^P_{A_i U}(x)} \\
 &\quad \exists^N_{\exists^P_{A_1 L}(x), \exists^P_{A_2 L}(x), \dots, \exists^P_{A_i L}(x)}, \quad \exists^N_{\exists^N_{A_1 U}(x), \exists^N_{A_2 U}(x), \dots, \exists^N_{A_i U}(x)} \\
 &\quad \exists^N_{\exists^N_{A_1 L}(x), \exists^N_{A_2 L}(x), \dots, \exists^N_{A_i L}(x)}, \quad \exists^N_{\exists^P_{A_1 U}(x), \exists^P_{A_2 U}(x), \dots, \exists^P_{A_i U}(x)} \\
 &\quad \exists^P_{\exists^P_{A_1 L}(x), \exists^P_{A_2 L}(x), \dots, \exists^P_{A_i L}(x)}, \quad \exists^P_{\exists^N_{A_1 U}(x), \exists^N_{A_2 U}(x), \dots, \exists^N_{A_i U}(x)} \\
 &\quad \exists^N_{\exists^P_{A_1 L}(x), \exists^P_{A_2 L}(x), \dots, \exists^P_{A_i L}(x)}, \quad \exists^N_{\exists^N_{A_1 U}(x), \exists^N_{A_2 U}(x), \dots, \exists^N_{A_i U}(x)}
 \end{aligned}$$

Hence the arbitrary union of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

iii.

$$A \square B = \exists^P_{\exists^P_{(A \square B)_L}(x), \exists^P_{(A \square B)_U}(x)}, \exists^P_{\exists^N_{(A \square B)_L}(x), \exists^N_{(A \square B)_U}(x)}, \exists^N_{\exists^P_{(A \square B)_L}(x), \exists^P_{(A \square B)_U}(x)}, \exists^N_{\exists^N_{(A \square B)_L}(x), \exists^N_{(A \square B)_U}(x)}$$

where

$$\exists^P_{(A \square B)_L}(x) = \max \left\{ \exists^P_{AL}(x), \exists^P_{BL}(x) \right\}$$

$$\exists^P_{(A \square B)_U}(x) = \min \left\{ \exists^P_{AU}(x), \exists^P_{BU}(x) \right\}$$

$$\exists^N_{(A \square B)_L}(x) = \min \left\{ \exists^N_{AL}(x), \exists^N_{BL}(x) \right\}$$

$$\exists^N_{(A \square B)_U}(x) = \max \left\{ \exists^N_{AU}(x), \exists^N_{BU}(x) \right\}$$

$$\exists^P_{(A \square B)_L}(x) = \max \left\{ \exists^P_{AL}(x), \exists^P_{BL}(x) \right\}$$

$$\exists^P_{(A \square B)_U}(x) = \min \left\{ \exists^P_{AU}(x), \exists^P_{BU}(x) \right\}$$

$$\exists^N_{(A \square B)_L}(x) = \min \left\{ \exists^N_{AL}(x), \exists^N_{BL}(x) \right\}$$

$$\mathbb{E}_{(A \square B)U}^N(x) = \max \left\{ \mathbb{E}_{AU}^N(x), \mathbb{E}_{BU}^N(x) \right\}$$

then

$$(\begin{bmatrix} A_1 & A_2 \end{bmatrix}) = \begin{cases} \begin{bmatrix} \mathbb{E}_{A_1L}^p(x), \mathbb{E}_{A_2L}^p(x) \\ \mathbb{E}_{A_1U}^p(x), \mathbb{E}_{A_2U}^p(x) \\ \mathbb{E}_{A_1L}^N(x), \mathbb{E}_{A_2L}^N(x) \\ \mathbb{E}_{A_1U}^N(x), \mathbb{E}_{A_2U}^N(x) \end{bmatrix}, & \text{if } x \in X \\ \begin{bmatrix} \mathbb{E}_{A_1L}^p(x), \mathbb{E}_{A_2L}^p(x) \\ \mathbb{E}_{A_1U}^p(x), \mathbb{E}_{A_2U}^p(x) \\ \mathbb{E}_{A_1L}^N(x), \mathbb{E}_{A_2L}^N(x) \\ \mathbb{E}_{A_1U}^N(x), \mathbb{E}_{A_2U}^N(x) \end{bmatrix}, & \text{if } x \notin X \end{cases}$$

where

$$\begin{aligned} \mathbb{E}_{A_1L}^p(x) &= \max \left\{ \mathbb{E}_{A_1L}^p(x), \mathbb{E}_{A_2L}^p(x) \right\} \\ \mathbb{E}_{A_1U}^p(x) &= \min \left\{ \mathbb{E}_{A_1U}^p(x), \mathbb{E}_{A_2U}^p(x) \right\} \\ \mathbb{E}_{A_1L}^N(x) &= \min \left\{ \mathbb{E}_{A_1L}^N(x), \mathbb{E}_{A_2L}^N(x) \right\} \\ \mathbb{E}_{A_1U}^N(x) &= \max \left\{ \mathbb{E}_{A_1U}^N(x), \mathbb{E}_{A_2U}^N(x) \right\} \\ \mathbb{E}_{A_2L}^p(x) &= \max \left\{ \mathbb{E}_{A_1L}^p(x), \mathbb{E}_{A_2L}^p(x) \right\} \\ \mathbb{E}_{A_2U}^p(x) &= \min \left\{ \mathbb{E}_{A_1U}^p(x), \mathbb{E}_{A_2U}^p(x) \right\} \\ \mathbb{E}_{A_2L}^N(x) &= \min \left\{ \mathbb{E}_{A_1L}^N(x), \mathbb{E}_{A_2L}^N(x) \right\} \\ \mathbb{E}_{A_2U}^N(x) &= \max \left\{ \mathbb{E}_{A_1U}^N(x), \mathbb{E}_{A_2U}^N(x) \right\} \end{aligned}$$

then

$$\begin{aligned} \mathbb{E}_{A_1L}^p(x) &= \max \left\{ \mathbb{E}_{A_1L}^p(x), \mathbb{E}_{A_2L}^p(x) \right\} \\ \mathbb{E}_{A_1U}^p(x) &= \min \left\{ \mathbb{E}_{A_1U}^p(x), \mathbb{E}_{A_2U}^p(x) \right\} \\ \mathbb{E}_{A_1L}^N(x) &= \min \left\{ \mathbb{E}_{A_1L}^N(x), \mathbb{E}_{A_2L}^N(x) \right\} \\ \mathbb{E}_{A_1U}^N(x) &= \max \left\{ \mathbb{E}_{A_1U}^N(x), \mathbb{E}_{A_2U}^N(x) \right\} \\ 1 \square \mathbb{E}_{A_1L}^p(x) &= \max \left\{ 1 \square \mathbb{E}_{A_1L}^p(x), 1 \square \mathbb{E}_{A_2L}^p(x) \right\} \\ 1 \square \mathbb{E}_{A_1U}^p(x) &= \min \left\{ 1 \square \mathbb{E}_{A_1U}^p(x), 1 \square \mathbb{E}_{A_2U}^p(x) \right\} \\ 1 \square \mathbb{E}_{A_1L}^N(x) &= \min \left\{ 1 \square \mathbb{E}_{A_1L}^N(x), 1 \square \mathbb{E}_{A_2L}^N(x) \right\} \end{aligned}$$

$$1 \square \hat{\square}^N_{([A_1 \square [A_2]_U]_U)}(x) = \max \left\{ 1 \square \hat{\square}^N_{A_1 U}(x), 1 \square \hat{\square}^N_{A_2 U}(x) \right\}$$

$$\begin{aligned} \square [] A_1 \square [] A_2 &= \left\{ \begin{array}{l} x, \hat{\square}^p_{([A_1 \square [A_2]_L]_U)}(x), \hat{\square}^p_{([A_1 \square [A_2]_U]_U)}(x), \\ \hat{\square}^N_{([A_1 \square [A_2]_L]_U)}(x), \hat{\square}^N_{([A_1 \square [A_2]_U]_U)}(x), \\ 1 \square \hat{\square}^p_{([A_1 \square [A_2]_L]_U)}(x), 1 \square \hat{\square}^p_{([A_1 \square [A_2]_U]_U)}(x), \\ 1 \square \hat{\square}^N_{([A_1 \square [A_2]_L]_U)}(x), 1 \square \hat{\square}^N_{([A_1 \square [A_2]_U]_U)}(x) \end{array} \right\} | x \square X \hat{\square}^N \\ \square [] A_1 \square [] A_2 \square \dots [] A_i &= \left\{ \begin{array}{l} x, \hat{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_L]_U)}(x), \hat{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_U]_U)}(x), \\ \hat{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_L]_U)}(x), \hat{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_U]_U)}(x), \\ \hat{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_L]_U)}(x), \hat{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_U]_U)}(x), \\ \hat{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_L]_U)}(x), \hat{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_U]_U)}(x) \end{array} \right\} | x \square X \end{aligned}$$

where

$$\hat{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_L]_U)}(x) = \max \left\{ \hat{\square}^p_{[A_1 L]}(x), \hat{\square}^p_{[A_2 L]}(x), \dots, \hat{\square}^p_{[A_i L]}(x) \right\}$$

$$\hat{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_U]_U)}(x) = \min \left\{ \hat{\square}^p_{[A_1 U]}(x), \hat{\square}^p_{[A_2 U]}(x), \dots, \hat{\square}^p_{[A_i U]}(x) \right\}$$

$$\hat{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_L]_U)}(x) = \min \left\{ \hat{\square}^N_{[A_1 L]}(x), \hat{\square}^N_{[A_2 L]}(x), \dots, \hat{\square}^N_{[A_i L]}(x) \right\}$$

$$\hat{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_U]_U)}(x) = \max \left\{ \hat{\square}^N_{[A_1 U]}(x), \hat{\square}^N_{[A_2 U]}(x), \dots, \hat{\square}^N_{[A_i U]}(x) \right\}$$

$$\hat{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_L]_U)}(x) = \max \left\{ \hat{\square}^p_{[A_1 L]}(x), \hat{\square}^p_{[A_2 L]}(x), \dots, \hat{\square}^p_{[A_i L]}(x) \right\}$$

$$\hat{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_U]_U)}(x) = \min \left\{ \hat{\square}^p_{[A_1 U]}(x), \hat{\square}^p_{[A_2 U]}(x), \dots, \hat{\square}^p_{[A_i U]}(x) \right\}$$

$$\hat{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_L]_U)}(x) = \min \left\{ \hat{\square}^N_{[A_1 L]}(x), \hat{\square}^N_{[A_2 L]}(x), \dots, \hat{\square}^N_{[A_i L]}(x) \right\}$$

$$\hat{\square}^N_{([A_1 \square [A_2 \square \dots [A_i]_U]_U)}(x) = \max \left\{ \hat{\square}^N_{[A_1 U]}(x), \hat{\square}^N_{[A_2 U]}(x), \dots, \hat{\square}^N_{[A_i U]}(x) \right\}$$

then

$$\hat{\square}^p_{([A_1 \square [A_2 \square \dots [A_i]_L]_U)}(x) = \max \left\{ \hat{\square}^p_{A_1 L}(x), \hat{\square}^p_{A_2 L}(x), \dots, \hat{\square}^p_{A_i L}(x) \right\}$$

$$\begin{aligned}
& \min_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] U\right)} \left(x \right) = \min \left\{ \min_{A_1 U} \left(x \right), \min_{A_2 U} \left(x \right), \dots, \min_{A_i U} \left(x \right) \right\} \\
& \min_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] L\right)} \left(x \right) = \min \left\{ \min_{A_1 L} \left(x \right), \min_{A_2 L} \left(x \right), \dots, \min_{A_i L} \left(x \right) \right\} \\
& \max_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] U\right)} \left(x \right) = \max \left\{ \max_{A_1 U} \left(x \right), \max_{A_2 U} \left(x \right), \dots, \max_{A_i U} \left(x \right) \right\} \\
& \max_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] L\right)} \left(x \right) = \max \left\{ \max_{A_1 L} \left(x \right), \max_{A_2 L} \left(x \right), \dots, \max_{A_i L} \left(x \right) \right\} \\
& \min_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] U\right)} \left(x \right) = \min \left\{ \min_{A_1 U} \left(x \right), \min_{A_2 U} \left(x \right), \dots, \min_{A_i U} \left(x \right) \right\} \\
& \min_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] L\right)} \left(x \right) = \min \left\{ \min_{A_1 L} \left(x \right), \min_{A_2 L} \left(x \right), \dots, \min_{A_i L} \left(x \right) \right\} \\
& \max_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] U\right)} \left(x \right) = \max \left\{ \max_{A_1 U} \left(x \right), \max_{A_2 U} \left(x \right), \dots, \max_{A_i U} \left(x \right) \right\} \\
& \max_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] L\right)} \left(x \right) = \max \left\{ \max_{A_1 L} \left(x \right), \max_{A_2 L} \left(x \right), \dots, \max_{A_i L} \left(x \right) \right\} \\
& \left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array} \right] A \right) = \left(\left[\begin{array}{c} x, \min_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] L\right)} \left(x \right), \min_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] U\right)} \left(x \right) \right], \left[\begin{array}{c} \min_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] L\right)} \left(x \right), \max_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] U\right)} \left(x \right) \right], \dots, \left(\left[\begin{array}{c} \min_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] L\right)} \left(x \right), \max_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] U\right)} \left(x \right) \right], \left[\begin{array}{c} x, \min_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] L\right)} \left(x \right), \min_{\left(\left[\begin{array}{c} A_1 \\ \vdots \\ A_i \end{array}\right] U\right)} \left(x \right) \right] \right) \right)
\end{aligned}$$

Hence the finite intersection of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

Hence the Bipolar Interval Valued Intuitionistic Necessity Operators itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

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